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Surface waves on the relativistic quantum plasma half-space

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ABSTRACT

We present a theoretical investigation on the propagation of surface waves on the relativistic quantum plasma half-space. The dispersion relations of surface plasmon polaritons (SPPs) and electrostatic surface waves containing relativistic quantum corrected terms are derived. Results show that the frequency of SPPs has a blue-shift, and surface Langmuir oscillations can propagate on the cold plasma half-space due to quantum effects. Numerical evaluation indicates that quantum effects to SPPs and electrostatic surface waves are significant and observable.

1. Introduction

The investigation on surface waves began with the pioneering theoretical works by Trivelpiece and Gould [1]. They restricted their attentions to electrostatic surface waves on cold cylindrical plasma columns. The effect of a finite temperature on the dispersion relation of electrostatic surface waves was studied by Ritchie [2], who applied a simple hydrodynamical model, and Guernsey [3], who adopted the kinetic theory. Vedenov provided a general dispersion relation without limiting to electrostatic approximation, which showed that in cold plasma half-space, surface plasma polaritons existed with frequencies ranging from $\omega_{spps} \to \omega_P/\sqrt{2}$ down to $\omega_{spps} = 0$ [4].

Early researches on surface waves are limited in the classical regime where quantum effects are neglected. However, high number density in dense plasmas (e.g., metallic plasmas, or intense laser-produced solid-density plasmas) and high degree of miniaturization of today's electronic components open up the possibilities that the quantum statistical effects should be included since the electron Fermi pressure is much larger than the thermal pressure. Additionally, the quantum tunneling effect can no longer be neglected due to the thermal de Broglie wavelength λ_B of electrons may be comparable to, or even larger than the average interparticle distance of electrons (viz., $\lambda_B^3 n_0 \geqslant 1$). Therefore, the behavior of surface waves on quantum plasma half-space has aroused wide attention in recent years. Relevant researches include Alfvén surface modes in a dusty electron–positron plasma with spin-1/2 effects [5], surface Langmuir oscillations in semi-bounded quantum plasmas [6], electrostatic/electromagnetic modes on the quantum plasma half-space [7], propagation of the transverse electric surface modes on semi-bounded quantum plasma in the presence of external magnetic field [8], and self-excited surface plasmon polaritons at the interface of counterstreaming plasmas [9].

The above researches are based on the non-relativistic quantum hydrodynamic model. It should be noticed that hydrodynamical model cannot effectively deal with the Landau damping caused by the resonance between waves and particles. Particularly, when the thermal velocity of particles approaches the phase velocity of wave, hydrodynamic model will be completely invalid. Besides, the Fermi velocity of electrons in dense plasmas is weak relativistic due to high number density. Therefore, relativistic effects should also be taken into account. Based on the above considerations, we will present a theoretical investigation on surface waves on the relativistic quantum plasmas half-space by adopting the relativistic quantum kinetic model, in which both quantum effects (the Fermi statistical pressure and quantum tunneling effects) and relativistic effects are included. The dispersion relations of SPPs and electrostatic surface waves on the relativistic quantum plasma half-space are presented, and the Landau damping of surface electrostatic waves is also derived. The relativistic quantum corrections are numerically evaluated with parameters of the next generation of laser-based plasma compression (LBPC) schemes.

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2. Dispersion relation and Landau damping of surface waves

The surface plasmon (SP)-polariton (SPP) is the collective electrostatic/electromagnetic excitation of electrons on the vacuum–plasma or plasma–plasma interface. The SPP propagates along the plasma interface and is evanescent in plasmas. For a plasma–vacuum interface, the SPP is described by the dispersion relation [10]:

$$\varepsilon\big(k^2c^2-\omega^2\big)^{1/2}=-\big(k^2c^2-\omega^2\varepsilon\big)^{1/2}, \tag{1}$$

where $\varepsilon(\omega,k)=1-\omega_p^2/\omega^2<0$ is the dielectric function of cold plasmas [11,12]. However, the dielectric function ε will be more complex for relativistic quantum plasmas.

In the past decades, relativistic quantum plasmas [13–15] have been widely investigated. A quantum plasmadynamics was established, and the relativistic quantum plasma dispersion functions were investigated by Melrose [16]. A covariant Wigner function approach for relativistic quantum plasmas was put forward by Hakim in 1978 [17], and the relativistic quantum theory was represented via the covariant Wigner function and Dirac equation. Based on the covariant Wigner function approach for relativistic quantum plasmas, the relativistic quantum correction to laser wakefield acceleration [18], and the dispersion relation and Landau damping of waves in high energy density plasmas were investigated [19].

The dielectric function ε for relativistic quantum plasmas [17,19] is

$$\varepsilon(\omega,k)=1-\frac{\Omega_p^2}{\omega^2}+\frac{\omega_p^2}{\omega^2}K^{11}, \tag{2}$$

where

$$\Omega_p^2=\frac{\omega_p^2}{n_0}\int d^4p\,f_0(p) \tag{3}$$

is the relativistic quantum plasma frequency,

$$K^{\lambda\sigma}=\frac{1}{\hbar n_0}\int d^4p\,\frac{p^\lambda p^\sigma}{k_\mu p^\mu}\left[f_0\left(p+\frac{1}{2}\hbar k\right)-f_0\left(p-\frac{1}{2}\hbar k\right)\right] \tag{4}$$

is the tensor of the dielectric permittivity. $\omega_p=(4\pi n_0e^2/m)^{1/2}$ is the plasma frequency, n_0 is the equilibrium density of electrons, and $f_0(p)$ is the equilibrium distribution of electrons.

Inserting Eq. (2) into Eq. (1), the dispersion relation of SPPs on relativistic quantum plasma half-space is obtained as

$$\frac{\omega^2}{k^2c^2}=1+\frac{\omega^2}{\omega^2-\Omega_p^2+\omega_p^2K^{11}}. \tag{5}$$

With the electrostatic limit $c\to\infty$, we get the dispersion relation of electrostatic surface waves

$$\omega^2=\frac{1}{2}\big(\Omega_p^2-\omega_p^2K^{11}\big). \tag{6}$$

The frequency of electrostatic surface waves can be decomposed as $\omega=\omega_r+i\gamma$, where ω_r and γ are the real and imaginary parts, respectively. Supposing $\gamma\ll\omega_r$, Eq. (6) can be decomposed as

$$\omega_r^2=\frac{1}{2}\big[\Omega_p^2-\omega_p^2\,\mathrm{Re}\big(K^{11}\big)\big], \tag{7}$$

and

$$\frac{\gamma_L}{\omega_r}=-\frac{\omega_p^2}{4\omega_r^2}\,\mathrm{Im}\big(K^{11}\big), \tag{8}$$

where γ_L/ω_r is the normalized Landau damping rate of the electrostatic surface waves.

In order to calculate K^{11}, we introduce the zero-temperature relativistic Fermi–Dirac distribution

$$f_0(p)=\frac{2mc^3}{(2\pi\hbar)^3}\int d^4p'\,\delta\big(p-p'\big)2\theta\big(p_0'c\big)\times\delta\big(p'^2c^2-m^2c^4\big)\theta\big(\epsilon_F-p_0'c\big), \tag{9}$$

where θ is the Heaviside step function, ϵ_F is the Fermi energy of electrons, and $p'=(p_0',\mathbf{p}')$. Inserting Eq. (9) into the expression of K^{11}, we have

$$K^{11}=\frac{mc^4}{4\pi^3\hbar^4n_0}\int d^3\mathbf{p}\times p_1^2\left[\frac{\theta(\epsilon_F-E_{p+\hbar k/2})}{E_{p+\hbar k/2}}\frac{1}{\omega(E_{p+\hbar k/2}-\frac{1}{2}\hbar\omega)-c^2\mathbf{k}\cdot\mathbf{p}}-\frac{\theta(\epsilon_F-E_{p-\hbar k/2})}{E_{p-\hbar k/2}}\frac{1}{\omega(E_{p-\hbar k/2}+\frac{1}{2}\hbar\omega)-c^2\mathbf{k}\cdot\mathbf{p}}\right], \tag{10}$$

where $E_p=(m^2c^4+p^2c^2)^{1/2}$. Performing the integration and taking weak relativistic approximation $p_F^2\ll m^2c^2$, we have

$$\mathrm{Re}\,K^{11}=\frac{\Omega_p^2}{\omega_p^2}-\left[1-\frac{1}{2}\beta_F^2+\frac{1}{5}\frac{k^2v_F^2}{\omega^2}+\frac{\hbar^2k^2v_F^2}{20m^2c^4}\left(\frac{k^4c^4}{\omega^4}-\frac{k^2c^2}{\omega^2}+1-\frac{\omega^2}{k^2c^2}\right)\right]\quad(v_F\ll v_{ph}) \tag{11}$$

and

$$\mathrm{Im}\,K^{11}=\frac{3\pi}{4}\frac{\omega}{kv_F}\left[1-\frac{\omega^2}{k^2v_F^2}\big(1+\beta_F^2\big)-\frac{\hbar^2k^2}{4m^2v_F^2}\right]\quad(v_F>v_{ph}), \tag{12}$$

where $\beta_F=v_F/c$, $v_F=(3\pi^2n_0)^{1/3}\hbar/m$ is the Fermi velocity of electrons and v_{ph} is the phase velocity of waves.

Inserting Eq. (11) into Eq. (5), we have

$$\frac{\omega^2}{k^2c^2}=1+\frac{\omega^2}{\omega^2-\omega_p^2[1-\frac{1}{2}\beta_F^2+\frac{1}{5}\frac{k^2v_F^2}{\omega^2}+\frac{\hbar^2k^2v_F^2}{20m^2c^4}(\frac{k^4c^4}{\omega^4}-\frac{k^2c^2}{\omega^2}+1-\frac{\omega^2}{k^2c^2})]}. \tag{13}$$

Without quantum effects, i.e., $\hbar\to0$, Eq. (13) is degenerated to the well-known dispersion relation of surface waves on the cold plasma half-space

$$\omega^2=\frac{1}{2}\omega_p^2+k^2c^2-\frac{1}{2}\sqrt{\omega_p^4+4k^4c^4}. \tag{14}$$

The electrostatic limit of Eq. (13) is taken by $c\to\infty$, as a result, we have the dispersion relation of electrostatic surface waves on the relativistic quantum plasma half-space

$$\omega_r^2=\frac{1}{2}\omega_p^2\left(1-\frac{1}{2}\beta_F^2\right)+\frac{1}{5}k^2v_F^2+\frac{\hbar^2k^4}{10m^2\omega_p^2}k^2v_F^2\quad(v_F\ll v_{ph}). \tag{15}$$

When $\hbar\to0$, Eq. (15) is degenerated to the well-known dispersion relation for the electrostatic surface waves on the cold plasma half-space. Eq. (15) indicates that surface Langmuir oscillations can propagate on the cold plasma half-space due to quantum effects.

The Landau damping of electrostatic surface waves on the relativistic quantum plasma half-space is

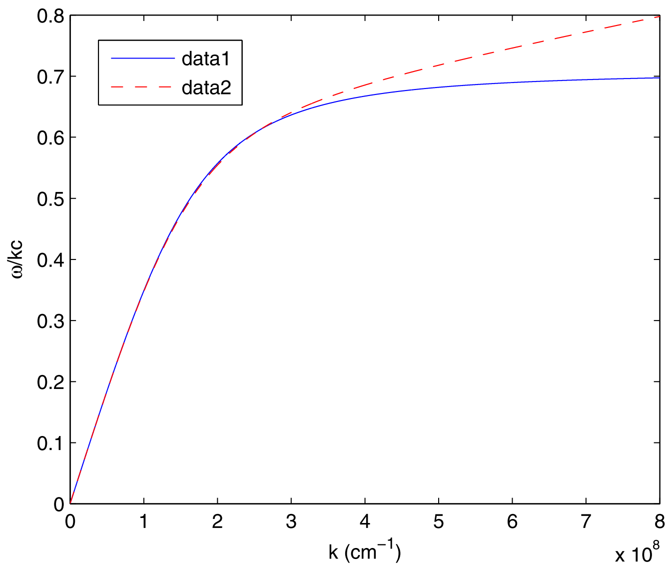
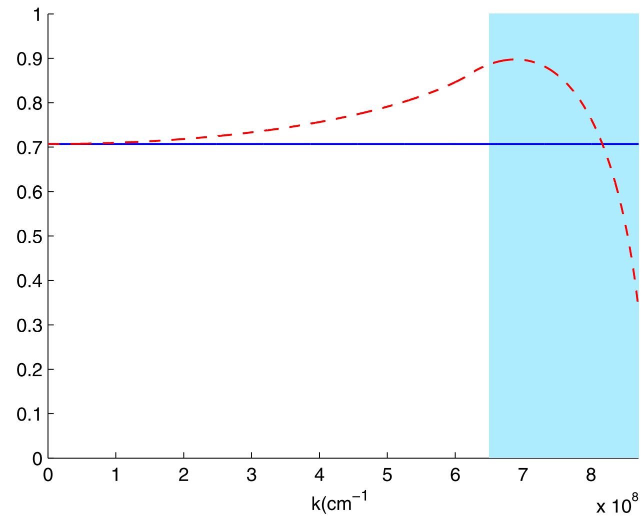


Fig. 1. Solid line: SPPs on the cold plasma half-space. Dashed line: SPPs on the relativistic quantum plasma half-space. $n_0 = 3 \times 10^{28}$ cm^{-3}, $\omega_p = 9.77 \times 10^{18}$ s^{-1}, and $v_F = 1.11 \times 10^{10}$ cm/s.



$$\gamma_L = -\frac{3\pi}{16}\frac{\omega_p^2}{k v_F}\left[1 - \frac{\omega_r^2}{k^2 v_F^2}\left(1+\beta_F^2\right) - \frac{\hbar^2 k^2}{4m^2 v_F^2}\right] \quad (v_F > v_{ph}). \tag{16}$$

3. Discussion and conclusion

It should be noticed that our theoretical model and results are only applicable to the weak relativistic quantum collisionless plasma systems. In this section, we adopt the parameters of LBPC schemes [20] for quantitative calculation. The parameters chosen are $n_0 = 3 \times 10^{28}$ cm^{-3}, $T = 10$ keV, $\omega_p = 9.77 \times 10^{18}$ s^{-1}, $v_F = 1.11 \times 10^{10}$ cm/s.

In quantum plasmas, since the de Broglie wavelength λ_B of electrons becomes comparable to, or even larger than the average interparticle distance of electrons (viz., $\lambda_B^3 n_0 \geqslant 1$), quantum effects are expected to play a crucial role in plasma dynamic. From the expression $\lambda_B^3 n_0 \geqslant 1$, we have

$$\frac{n_0}{T^{3/2}} \geqslant \left(\frac{\sqrt{mk_B}}{\hbar}\right)^3 \sim 10^{16}\ \text{cm}^{-3}/\text{K}^{3/2}. \tag{17}$$

Obviously, the parameters of LPBC schemes satisfy the quantum condition $\lambda_B^3 n_0 \geqslant 1$ and the weak relativistic condition $v_F^2/c^2 \ll 1$.

The quantum coupling parameter can be defined as the ratio of the interaction energy E_{int} to the average kinetic energy E_{kin}. The interaction energy of electrons situated at typical interparticle distance $d = n_0^{-1/3}$ is $E_{int} = e^2/(\varepsilon_0 d)$, and the kinetic energy is given by the Fermi energy $E_{kin} = E_F$. The relativistic Fermi energy is defined as

$$E_F = mc^2\left(\frac{1}{\sqrt{1 - v_F^2/c^2}} - 1\right). \tag{18}$$

With weak relativistic approximation $v_F^2/c^2 \ll 1$, the relativistic Fermi energy can be expressed as

$$\begin{aligned} E_F &= mc^2\left(1 + \frac{v_F^2}{2c^2} + \frac{3v_F^4}{8c^4} + \cdots - 1\right) \\ &\simeq \frac{1}{2} m v_F^2\left(1 + \frac{3}{4}\frac{v_f^2}{c^2}\right). \end{aligned} \tag{19}$$

responding Landau damping curve of electrostatic surface waves is shown in Fig. 3.

To summarize, we present a theoretical investigation on the propagation of surface waves on the relativistic quantum plasma half-space by applying the relativistic quantum kinetic model. The dispersion relations of SPPs and electrostatic surface waves are derived respectively. Research shows that the frequency of SPPs has a blue-shift, and surface Langmuir oscillations can propagate on the cold plasma half-space due to quantum tunneling effects and Fermi statistical pressure. Numerical evaluation indicates that quantum effects to the dispersion relation of SPPs and electrostatic surface waves are significant and observable in LBPC schemes.

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