

**Supplementary Note 1: Spatial transfer function for surface plasmon polariton (SPP) excitations at a metallic surface**

To analyze the the spatial transfer function, we developed the spatial coupled-mode theory to describe the SPP excitation at a metallic surface. As Supplementary Fig. 1 shows, in the Kretschmann configuration, a  $p$ -polarized incident beam illuminates the metal layer coating on a glass prism. When the parallel component of the incident wavevector,  $\tilde{k}_z$ , is close to the SPP wavevector  $\beta_{\text{spp}}$ , the incident light excites an SPP through an evanescent wave. Meanwhile the excited SPP leaks out and generates the radiation wave in the glass as it propagates along the  $\tilde{z}$  direction. Therefore, the reflection process consists of two pathways: the direct reflectance of the incident wave at the glass-metal interface, and the outgoing radiation from the leakage of the excited SPP at the metal-air interface. Based on the spatial coupled-mode theory [1, 2], the spatial mode coupling and the interference process is described by the following equations:

$$\frac{da}{d\tilde{z}} = (i\beta_{\text{spp}} - \alpha_1 - \alpha_{\text{spp}})a + ie^{i\phi/2}\sqrt{2\alpha_1}\tilde{S}_{\text{in}}(\tilde{z}) \quad (1)$$

$$\tilde{S}_{\text{out}} = e^{i\phi}\tilde{S}_{\text{in}} + ie^{i\phi/2}\sqrt{2\alpha_1}a, \quad (2)$$

Here, we take the convention that the field varies in time as  $\exp(-i\omega t)$ .  $a$  is the amplitude of the SPP which is normalized such that  $|a|^2$  corresponds to the time-averaged power along the  $\tilde{z}$  direction. We note that  $\tilde{S}_{\text{in}}$  ( $\tilde{S}_{\text{out}}$ ) corresponds to the field distribution of the incident (reflected) light along the  $\tilde{z}$  direction. Therefore, by expanding the incident (reflected) field into a series of plane waves as  $\tilde{S}_{\text{in(out)}} = \int_{-\infty}^{\infty} \tilde{s}_{\text{in(out)}}(\tilde{k}_z) \exp(i\tilde{k}_z\tilde{z})d\tilde{k}_z$ , the reflection coefficient for the incident plane wave case with a wavevector component  $\tilde{k}_z$  is obtained as [1]

$$R(\tilde{k}_z) \equiv \frac{\tilde{s}_-}{\tilde{s}_+} = e^{i\phi} \frac{i(\tilde{k}_z - \beta_{\text{spp}}) - \alpha_1 + \alpha_{\text{spp}}}{i(\tilde{k}_z - \beta_{\text{spp}}) + \alpha_1 + \alpha_{\text{spp}}}. \quad (3)$$

We note that the  $\tilde{z}$  coordinate defined on the metal surface is rather different from the  $x$  direction of the incident (reflected) beam profile coordinate, which is defined as perpendicular to the beam propagation direction. For the incident angle  $\theta_0$ , the plane wave with the wavevector component  $k_x$  at the  $x$  direction has a  $\tilde{z}$  component [3, 4]

$$\tilde{k}_z = k_x \cos \theta_0 + \beta_{\text{spp}}, \quad \left| \tilde{k}_z \right| < w, \quad (4)$$

where  $w$  is the angle spectrum width and assumed as  $w \cos \theta_0 < \alpha_1 + |\alpha_{\text{spp}}|$ . Supplementary Eq. (4) can be understood as the central frequency of the angle spectrum shifting from 0

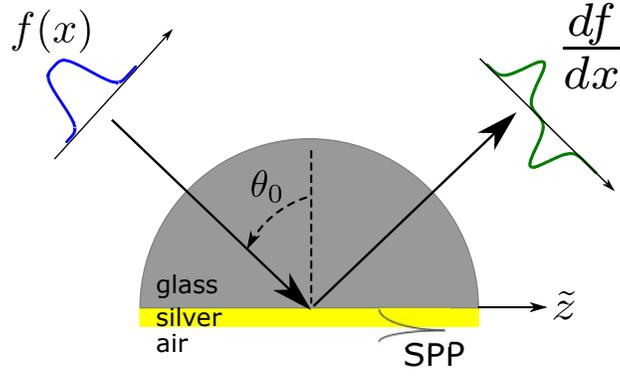
to  $\beta_{\text{spp}}$ . Such a spatial frequency shifting also happens to the reflected beam. Therefore, by substituting Supplementary Eq. (4) into (3), we obtain the transfer function during the reflection:

$$\begin{aligned} H(k_x) &= R(k_x \cos \theta_0 + \beta_{\text{spp}}) \\ &= e^{i\varphi} \frac{ik_x + A}{ik_x + B}, \end{aligned} \quad (5)$$

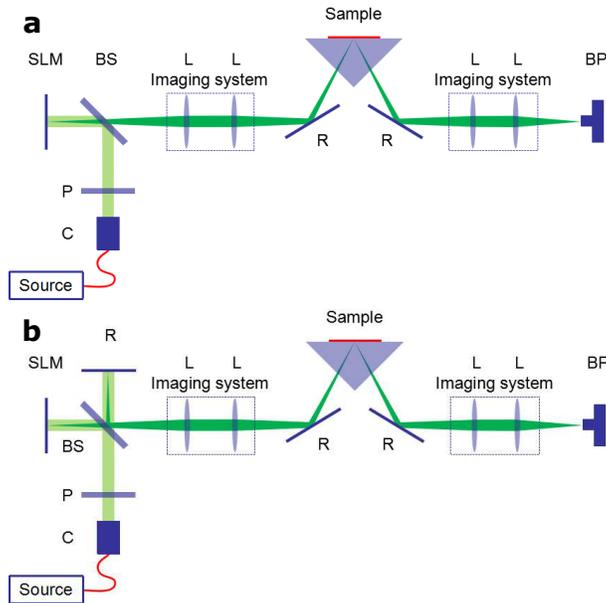
where  $A = (\alpha_{\text{spp}} - \alpha_1)/\cos \theta_0$  and  $B = (\alpha_{\text{spp}} + \alpha_1)/\cos \theta_0$ .

### **Supplementary Note 2: Phase/Amplitude modulated field image generated by spatial light modulator**

We generate the incident field images by a spatial light modulator (SLM: Holoeye PLUTO-NIR-011), which is a reflective-phase-only modulator. For the phase modulation, the system setup is schematically shown in Supplementary Fig. 2a, where a collimated laser beam was expanded and illuminated on the SLM and the phase-modulated image field was projected onto the metal film with a conjugate imaging system. In order to generate the amplitude-modulated incident field by the SLM, we used a Michelson configuration to make the phase-modulated field interfere with a reference plane wave (Supplementary Fig. 2b).



**Supplementary Fig. 1. Schematic of a SPP reflector with the Kretschmann configuration** Here,  $\theta_0$  corresponds to the incident angle where the phase-matching condition for SPP excitation is satisfied.



**Supplementary Fig. 2. Schematic of optical system for the phase- and amplitude-modulation** (a) and (b) correspond to phase- and amplitude-modulation, respectively. The field image is generated by spatial light modulator (SLM). Components include C: collimator, P: polarizer, BS: beam splitter, L: lens, SLM: spatial light modulator, R: reflector, BP: beam profiler.

## Supplementary References

- [1] Ruan, Z., Wu, H., Qiu, M. & Fan, S. Spatial control of surface plasmon polariton excitation at planar metal surface. *Opt. Lett.* **39**, 3587–3590 (2014).
- [2] Lou, Y., Pan, H., Zhu, T. & Ruan, Z. Spatial coupled-mode theory for surface plasmon polariton excitation at metallic gratings. *J. Opt. Soc. Am. B* **33**, 819–824 (2016).
- [3] Goodman, J. *Introduction to Fourier optics* 2nd edn (McGraw-hill, 1996).
- [4] Doskolovich, L. L., Bykov, D. A., Bezus, E. A. & Soifer, V. A. Spatial differentiation of optical beams using phase-shifted Bragg grating. *Opt. Lett.* **39**, 1278–1281 (2014).